CSC 611 – Algorithms and Advanced Data Structures
Exam #6, Fall 2024

First/Given Name: _____

Last/Family Name: _____

This exam contains 4 pages (including this cover page) and 3 questions.

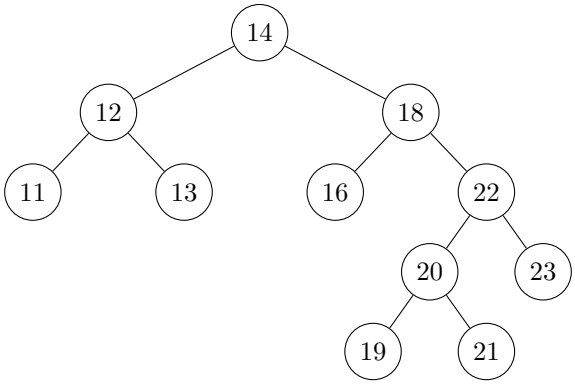
- Clearly identify your answer for each problem, and try to organize your work in a reasonably coherent way, in the space provided. If you decided to use the back of a paper, note this clearly so the instructor can find your answer. You can simplify and shortify answers by combining terms, reducing fractions, etc, to an extent that it still shows you understand what are you doing.
- It might be a good idea to draw a box around your final answer.
- Partial credit will be given for incorrect answers that show a partial understanding of the relevant concepts. Therefore, it is a good idea to show your work to convince your instructor that you understand the material. Irrelevant and meaningless answers will not receive partial credit.
- No electronic devices, including calculators, are allowed.
- You have 30 minutes to complete this exam.
- Each student is allowed to use a cheat sheet of size $4.5'' \times 5.75''$, which is equivalent to one-fourth of a standard letter-sized paper. The cheat sheet can be used on both sides. Only hand-written cheat sheets are allowed, and each student is required to write their name on their cheat sheet. The cheat sheet must be submitted along with the exam upon completion.

Question	Points	Score
1	2.00	
2	2.00	
3	2.00	
Total:	6.00	

I acknowledge that it is the responsibility of every student at Missouri State University to adhere to the university's policies on Student Academic Integrity. I confirm that I have neither given nor received any unauthorized assistance during this exam.

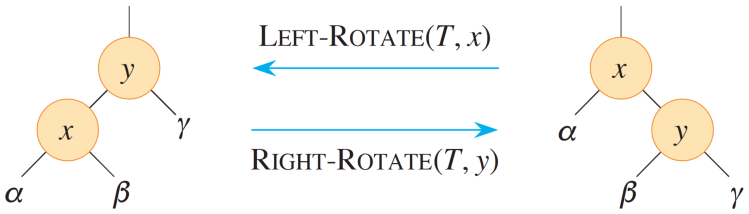
Signature: _____

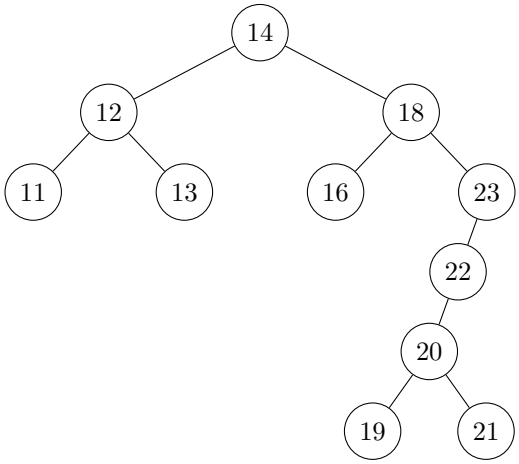
1. (2.00 points) Assume in the following binary search tree, x is the node with key 22. Draw the resulting tree after executing LEF-ROTATE(T, x).



Solution:

Considering the following rules for left rotation and right rotation, y is node with key 23, α is the subtree rooted at node with key 20, and β and γ are both empty subtrees. Thus, the solution is as follows.





 Points earned for this question: _____

2. Given four matrices A_1, A_2, A_3, and A_4 with the dimensions shown below, we want to compute the matrix chain multiplication $A_1A_2A_3A_4$ in and order that minimizes the number of scalar multiplications. Using the dynamic programming algorithm, the tables m and s are partially filled in as below.

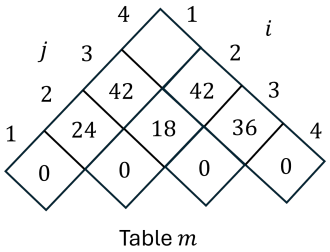


Table m

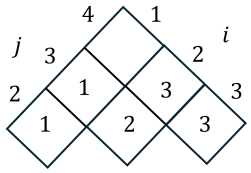


Table s

Matrix	A_1	A_2	A_3	A_4
Dimension	4×2	2×3	3×3	3×4

$$p_0 = 4, p_1 = 2, p_2 = 3, p_3 = 3, p_4 = 4 \tag{1}$$

(a) (1.00 points) Compute the values $m[1, 4]$ and $s[1, 4]$.

Solution:

$$notagm[1,4] = \min \begin{cases} m[1,1] + m[2,4] + p_0p_1p_4 = 0 + 42 + 4 \cdot 2 \cdot 4 = 74, \\ m[1,2] + m[3,4] + p_0p_2p_4 = 24 + 36 + 4 \cdot 3 \cdot 4 = 108, \\ m[1,3] + m[4,4] + p_0p_3p_4 = 42 + 0 + 4 \cdot 3 \cdot 4 = 90 \end{cases}$$
$$= 74.$$

$$\Rightarrow s[1,4] = \arg \min_{1 \le k < 4} (m[1,k] + m[k+1,4] + p_0p_kp_4) = 1.$$

(b) (1.00 points) Using these tables, find the optimal parenthesization for $A_1A_2A_3A_4$.

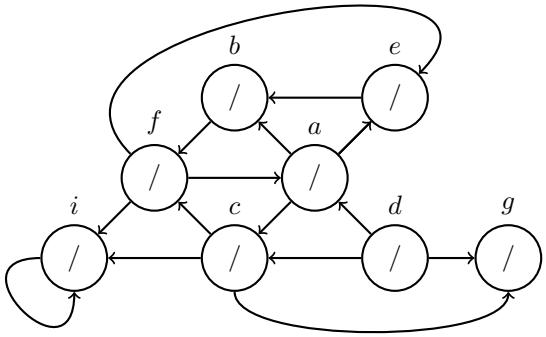
Solution: We use PRINT-OPTIMAL-PARENS (Page 381). For simplicity, we use PPP to denote PRINT-OPTIMAL-PARENS.

- PPP$(s, 1, 4)$:
  - print (
  - PPP$(s, 1, s[1, 4])$ = PPP$(s, 1, 1)$: Print A_1
  - PPP$(s, s[1, 4] + 1, 4)$ = PPP$(s, 2, 4)$:
    - print (
    - PPP$(s, 2, s[2, 4])$ = PPP$(s, 2, 3)$:
      - print (
      - PPP$(s, 2, s[2, 3])$ = PPP$(s, 2, 3)$: print A_2
      - PPP$(s, s[2, 3] + 1)$ = PPP$(s, 3, 3)$: print A_3
      - print)
    - PPP$(s, s[2, 4] + 1, 4)$ = PPP$(s, 4, 4)$: print A_4
    - print)
  - print)

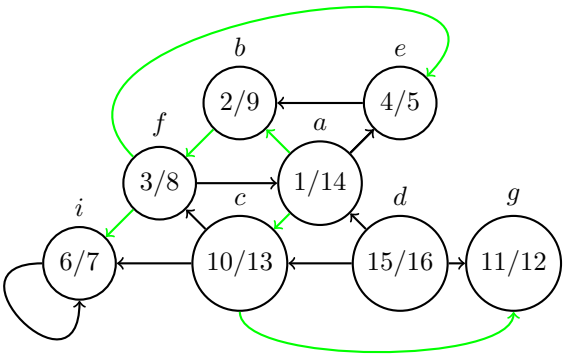
Accordingly, the optimal parenthesization is $A_1((A_2A_3)A_4)$.

 Points earned for this question: _____

3. (2.00 points) Execute the depth-first search (DFS) on the following graph and write the discovery time and finish time of each node. You can write those numbers inside the circles. Also, indicate which edges belong to the DFS tree. Assume that the graph is represented by adjacency lists and that the nodes within each list are stored in alphabetical order. The list of vertices is also stored in alphabetical order.



Solution:



 Points earned for this question: _____