CSC 611 – Algorithms and Advanced Data Strcutures
Exam #4, Fall 2024

First/Given Name: _____

Last/Family Name: _____

This exam contains 4 pages (including this cover page) and 3 questions.

- Clearly identify your answer for each problem, and try to organize your work in a reasonably coherent way, in the space provided. If you decided to use the back of a paper, note this clearly so the instructor can find your answer. You can simplify and shortify answers by combining terms, reducing fractions, etc, to an extent that it still shows you understand what are you doing.

- It might be a good idea to draw a box around your final answer.

- Partial credit will be given for incorrect answers that show a partial understanding of the relevant concepts. Therefore, it is a good idea to show your work to convince your instructor that you understand the material. Irrelevant and meaningless answers will not receive partial credit.

- No electronic devices, including calculators, are allowed.

- You have 30 minutes to complete this exam.

- Each student is allowed to use a cheat sheet of size $4.5'' \times 5.75''$, which is equivalent to one-fourth of a standard letter-sized paper. The cheat sheet can be used on both sides. Only hand-written cheat sheets are allowed, and each student is required to write their name on their cheat sheet. The cheat sheet must be submitted along with the exam upon completion.

Question	Points	Score
1	2.00	
2	2.00	
3	2.00	
Total:	6.00	

I acknowledge that it is the responsibility of every student at Missouri State University to adhere to the university's policies on Student Academic Integrity. I confirm that I have neither given nor received any unauthorized assistance during this exam.

Signature: _____

1. (2.00 points) Illustrate the operations of PARTITION, which was used in quicksort, on the following array:

$$A = \langle 4 \quad 7 \quad 1 \quad 8 \quad 2 \quad 5 \quad 3 \quad 6 \rangle.$$

Solution: The pivot is $x = A[r] = 6$

- i p, j r

  | 4 | 7 | 1 | 8 | 2 | 5 | 3 | 6 |

  $A[j] \leq x \Rightarrow i = 1$, swap $A[i]$ and $A[j], j = 2$

- p, i j r

  | 4 | 7 | 1 | 8 | 2 | 5 | 3 | 6 |

  $A[j] > x \Rightarrow j = 3$

- p, i j r

  | 4 | 7 | 1 | 8 | 2 | 5 | 3 | 6 |

  $A[j] \leq x \Rightarrow i = 2$, swap $A[i]$ and $A[j], j = 4$

- p i j r

  | 4 | 1 | 7 | 8 | 2 | 5 | 3 | 6 |

  $A[j] > x \Rightarrow j = 5$

- p i j r

  | 4 | 1 | 7 | 8 | 2 | 5 | 3 | 6 |

  $A[j] \leq x \Rightarrow i = 3$, swap $A[i]$ and $A[j], j = 6$

- p i j r

  | 4 | 1 | 2 | 8 | 7 | 5 | 3 | 6 |

  $A[j] \leq x \Rightarrow i = 4$, swap $A[i]$ and $A[j], j = 7$

- p i j r

  | 4 | 1 | 2 | 5 | 7 | 8 | 3 | 6 |

  $A[j] \leq x \Rightarrow i = 5$, swap $A[i]$ and $A[j], j = 8$

- p i r, j

  | 4 | 1 | 2 | 5 | 3 | 8 | 7 | 6 |

  $j = r \Rightarrow$ swap $A[i + 1]$ and $A[r]$

- p i r, j

  | 4 | 1 | 2 | 5 | 3 | 6 | 7 | 8 |

         Points earned for this question: _____

2. (2.00 points) Consider the following randomized algorithm.

RANDOMIZEDALGC(n)

1    $k = $ random integer between 1 and n^2
2    $c = 0$
3   **for** $i = 1$ to n
4   **for** $j = 1$ to k
5           $c = c + 1$
6   **return** c

Use probabilistic analysis to compute the expected running time of this algorithm in terms of Θ.

Solution:

$$
\begin{aligned}
E(n) &= \frac{1}{n^2} \sum_{k=1}^{n^2} \sum_{i=1}^{n} \sum_{j=1}^{k} 1 \\
&= \frac{1}{n^2} \sum_{k=1}^{n^2} \sum_{i=1}^{n} k \\
&= \frac{1}{n^2} \sum_{k=1}^{n^2} (nk) \\
&= \frac{1}{n} \sum_{k=1}^{n^2} k \\
&= \frac{1}{n} \cdot \frac{n^2(n^2+1)}{2} \\
&= \frac{n(n^2+1)}{2} \\
&= \frac{1}{2}n^3 + \frac{1}{2}n \\
&= \Theta(n^3).
\end{aligned}
$$

       Points earned for this question: _____

3. (2.00 points) Draw the decision tree that uses comparisons $(<, <=, >, >=, =)$ for sorting an array $\langle a_1, a_2, a_3 \rangle$. How many leaf nodes this tree has?
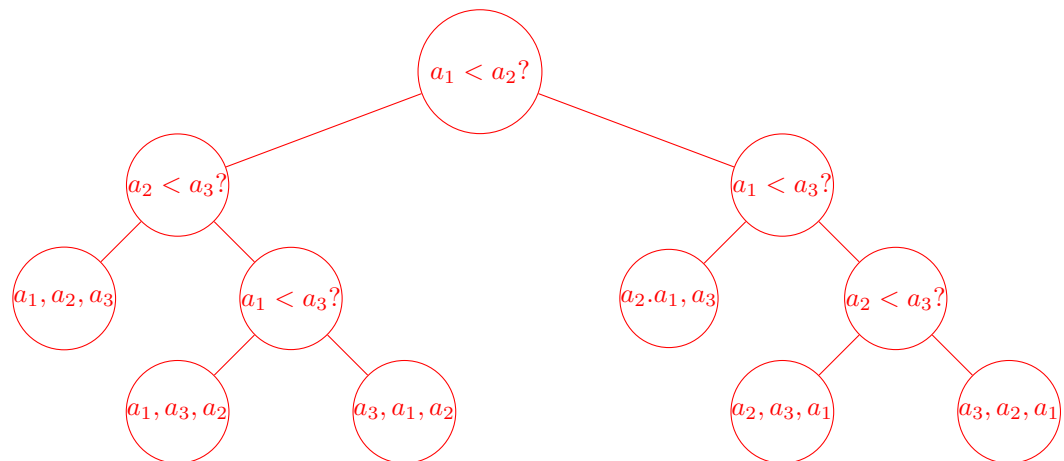
Solution:
The left branches are for Yes, and the right branches are for No.

Points earned for this question: _____